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Analysis of Laminar Flow through a Porous Channel with Velocity Slip

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ABSTRACT

The present paper analyses the problem of laminar flow in a porous channel with velocity slip using novel Computer Extended Series (CES) and Homotopy Analysis Method (HAM). The semi-numerical scheme described here offer some advantages over solution obtained by using traditional methods such as regular perturbation, shooting method etc. These techniques also reveal the analytic structure of the solution function. The objective is to study the influence of non-zero tangential slip velocity on velocity field and pressure gradient. Domb-Syke plot and h-curves enable us in obtaining the domain and rate of convergence of the series generated, which are further increased by Padé approximants. The solution presented here is valid for much larger Reynolds number for different derived quantities compared with earlier findings by Singh and Laurence (1979).

Keywords: Computer Extended Series, Homotopy Analysis Method, Velocity slip coefficient, Domb-Syke plot, $h-$curve.

1. Introduction

The problem of fluid flow in channels and tubes have received considerable attention in recent years, owing to its application in biological and engineering problems. Berman (1953) was the first researcher who studied the problem of steady flow of an incompressible viscous fluid through a porous channel with rectangular cross-section, when the Reynolds number is low. Yuan (1956) extended the problem of two dimensional steady state laminar flow in channels with porous walls for the case of various values of suction and injection Reynolds numbers. Sellars (1955) extended the problem studied by Berman for large positive Reynolds number.

Later Yuan (1956) extended the same problem for large negative values of Reynolds number. Terrill (1964) extended Berman's problem and obtained a more accurate numerical solution for fourth order non-linear differential equation. Following researchers Terrill (1969), Brady (1984), Robinson (1976), Cox (1991), King and Cox (2001) have extended Berman's problem and obtained the solution for large values of suction and injection. In the previous analysis of flow in porous channels majority have used no slip boundary conditions.

The experiments reported by Beavers and Joseph (1967) proved the existence of slip velocity at porous boundaries. The historical background to Beavers-Joseph conditions at the interface of porous media and clear fluid were reported by Nield (2009). Singh and Laurence (1979) obtained analytic expression for the velocity profile and pressure drop in the study of laminar flow through porous channels for small values of Reynolds number.

In this manuscript we re-investigate the problem of laminar flow in a porous channel with velocity slip (Singh and Laurence (1979)). In the present work we attempt to study the effect of non-zero tangential slip coefficient on the velocity field, pressure gradient using two novel techniques CES and HAM and present some useful, interesting results.

In the first method, we use Computer extended series (CES) method to solve the governing equations for moderately large wall Reynolds number R and generate large number of universal polynomial coefficient functions using MATHEMATICA and Domb-Syke's plot reveals convergence of the series. Van Dyke (1974, 1975, 1984) pioneered the use of computer extended series analysis in computational fluid dynamics. In an earlier study Bujurke et al. (2005, 1996) also successfully used this method. In the second method, we employed Homotopy Analysis Method for the solution of governing equation. Liao (1992) proposed the general analytic method for the solution of non-linear differential equations. Siddheshwar (2010) has used this technique to solve Ginzburg-Landau Equation with a time periodic coefficient. HAM provides an efficient solution with high accuracy and unlike perturbation method it is independent of very small or large physical parameters. The domain and rate of convergence is found by a proper choice of the auxiliary parameter h using $h-$curve. Finally, Padé approximants of various order give converging sum for sufficiently large Reynolds number.

An outline of the rest of this paper is as follows. In section 2 a brief mathematical formulation of the proposed problem is explained. Section 3 is devoted to approximate solution of the problem by Computer Extended series and Homotopy analysis method and in section 4, we compare the results obtained and discussed the influence of slip coefficient on velocity profiles and pressure gradient for different Reynolds number.

2. Mathematical Formulation

Consider a laminar flow of an incompressible viscous fluid between two plane parallel porous boundaries. It is assumed that the width of the channel is very large relative to height, thus the flow is assumed to be two dimensional and steady. We choose a Cartesian coordinate system (x, y) where x axis is in a plane parallel to channel walls and y axis is perpendicular it(Fig.1). The distance between wall is taken to be $2h$ and channel length is L. The flow is symmetrical about the mid plane of channel of half thickness h. Under the assumed condition and choice of axes, the relevant equations of linear momentum (Navier -Stokes equations) and continuity are,

$$u\frac{\partial u}{\partial x} + \frac{v}{h}\frac{\partial u}{\partial \lambda} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{h^2}\frac{\partial^2 u}{\partial \lambda^2}\right) \tag{1}$$

$$u\frac{\partial v}{\partial x} + \frac{v}{h}\frac{\partial v}{\partial \lambda} = -\frac{1}{\rho h}\frac{\partial p}{\partial \lambda} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{1}{h^2}\frac{\partial^2 v}{\partial \lambda^2}\right) \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{1}{h}\frac{\partial v}{\partial \lambda} = 0 \tag{3}$$

where $\lambda = \frac{y}{h}$ is the dimensionless variable.

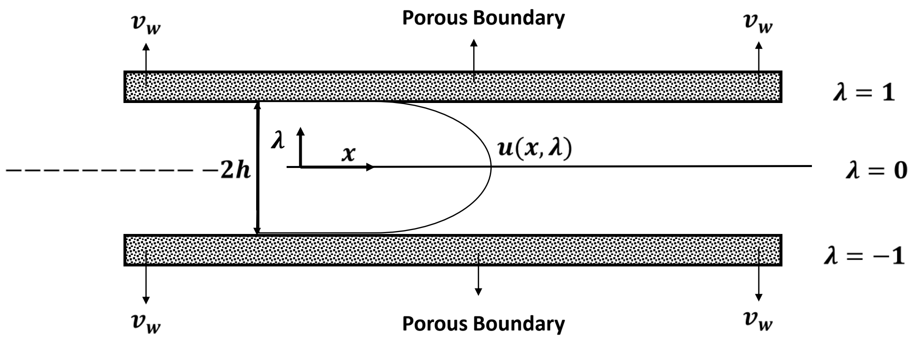


Figure 1: Schematic diagram of the problem

The requisite boundary conditions are,

$$
\begin{aligned}
u(x, \pm 1) &= -\frac{k^{1/2}}{\alpha h}\left(\frac{\partial u}{\partial \lambda}\right), \\
\left(\frac{\partial u}{\partial \lambda}\right)_{\lambda=0} &= 0, \\
v(x, 0) &= 0, \\
v(x, \pm 1) &= v_w = \text{constant}.
\end{aligned} \tag{4}
$$

The slip coefficient is given by relation $\phi = \frac{k^{1/2}}{\alpha h}$ α is a dimensionless constant which depends on the porous membrane and k is permeability. Following the procedure of Berman (1953), for constant wall velocity v_w, a suitable stream function ψ is chosen as,

$$\psi(x, \lambda) = [h\bar{u}(0) - v_w x]\, f(\lambda)$$

where $U(0)$ is an arbitrary velocity at $x = 0$.

The velocity components are given by

$$u(x, \lambda) = \frac{1}{h}[h\bar{u}(0) - v_w x]\, f'(\lambda) \tag{5}$$

$$v(\lambda) = v_w f(\lambda) \tag{6}$$

In these equations $f(\lambda)$, is some function of the distance parameter λ, which is to be determined.

Substituting Eq.(5) and Eq.(6) into the equations of motion Eq.(1) and Eq.(2), results in following equations,

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \left[\bar{u}(0) - \frac{v_w x}{h}\right]\left\{-\frac{v_w}{h}[f'^2 - ff''] - \frac{v}{h^2}f'''\right\}, \tag{7}$$

$$-\frac{1}{h\rho}\frac{\partial p}{\partial \lambda} \quad = \quad \frac{v}{h}\frac{\partial v}{\partial \lambda} - \frac{v}{h^2}\frac{\partial^2 v}{\partial \lambda^2}. \tag{8}$$

On eliminating the pressure p and simplifying, we get

$$\frac{d}{d\lambda}\left\{-\frac{v_w}{h}[f'^2 - ff''] - \frac{v}{h^2}f'''\right\} = 0.$$

After differentiation, we have,

$$f'''' + R(f'f'' - ff''') = 0. \tag{9}$$

The new boundary conditions are

$$\begin{aligned} f'(1) &= -\phi f''(1); \\ f''(0) &= 0 \\ f(0) &= 0; \\ f(1) &= 1. \end{aligned} \tag{10}$$

Eq.(9) is one of the Falkner-Skan family of equations. Analytical solutions for homogeneous counterpart of classical Falkner-Skan equation have been found by many authors Yang and Chien (1975), Brauner et al. (1982), Sachdev et al. (2008), Kudenatti et al. (2017). Eq. (9) along with boundary condition (10) is solved by a first order regular perturbation method by Singh and Laurence (1979), which is valid only for small R. This approach fails for any arbitrary R. The proposed series solution offers an attractive alternative approach and also the terms of this series method are capable of providing results to any desired degree accuracy for any moderately arbitrary R.

3. Method of Solution

3.1 Computer Extended Series

For the analysis of perturbation series we need sufficiently large number of coefficients in polynomial functions. Manually it is difficult to calculate beyond first order term as it involves complex algebraic calculations. Towards this goal, we consider solution of Eq.(9) in power series of R as,

$$f(\lambda) = f_0(\lambda) + \sum_{n=1}^{\infty} R^n f_n(\lambda). \tag{11}$$

Substituting (11) in (9) and equating various powers of R on both sides, we get,

$$f_n^{''''} = \Big(\sum_{r=0}^{n-1} f_r f_{n-1-r}''' - f_r' f_{n-1-r}''\Big), \quad n = 1, 2, 3, ... \tag{12}$$

and the relevant boundary conditions are,

$$\begin{aligned} &f_0(1) = 1, \\ &f_n'(1) = -\phi f_n''(1), \\ &f_n''(0) = f_n(0) = 0, \\ &f_n(1) = 0, \quad \text{for } n \geq 1. \end{aligned} \tag{13}$$

We solve Eq.(12) recursively using MATHEMATICA and able to generate universal polynomial functions $(f_n(\lambda), n = 0, 1, 2, 3, \ldots, 30)$ for different values of slip coefficient ϕ.

The solutions to the above equations, up to the term in R are (Singh and Laurence (1979)):

$$\begin{aligned} f_0(\lambda) &= -\frac{0.5\lambda^3}{3\phi+1} + \frac{3\lambda\phi}{3\phi+1} + \frac{1.5\lambda}{3\phi+1}, \\ f_1(\lambda) &= -\frac{0.0107143\lambda^7\phi}{(3\phi+1)^3} - \frac{0.00357143\lambda^7}{(3\phi+1)^3} + \frac{0.075\lambda^3\phi}{(3\phi+1)^3} + \frac{0.0107143\lambda^3}{(3\phi+1)^3} \\ &\quad - \frac{0.0642857\lambda\phi}{(3\phi+1)^3} - \frac{0.00714286\lambda}{(3\phi+1)^3} \end{aligned}$$

Velocity profiles:

From Eq. 5 and 6, expressions for velocity profile in the axial and transverse directions are,

$$U = \frac{u}{\bar{u_0}} = \Big(1 - \frac{4R}{Re}\frac{x}{h}\Big)\Big[f_0'(\lambda) + \sum_{n=1}^{\infty} R^n f_n'(\lambda)\Big] \tag{14}$$

$$V = \frac{v}{v_w} = f_0(\lambda) + \sum_{n=1}^{\infty} R^n f_n(\lambda) \tag{15}$$

Also, we have obtained the Normalized axial velocity component as,

$$u_s = \frac{u}{\bar{u}} = f'(\lambda) = \Big[f_0'(\lambda) + \sum_{n=1}^{\infty} R^n f_n'(\lambda)\Big] \tag{16}$$

For Normalized slip velocity we have,

$$u_s^0 = \left(\frac{u}{\bar{u}}\right)_{y=h} \tag{17}$$

Pressure gradient:

From (5, 6) and (9), an expression for Normalized pressure gradient P along the channel length is derived as,

$$P = \frac{-2C_P}{Re}\left[\frac{x}{h} - \frac{2R}{Re}\left(\frac{x}{h}\right)^2\right] \tag{18}$$

where C_P(integrating constant) obtained as,

$$C_P = f'''(0) = \sum_{n=0}^{\infty} R^n f_n'''(0) = \sum_{n=0}^{\infty} R^n a_n \tag{19}$$

Coefficients of the above series (14) and (18) are decreasing in magnitude. We used Domb-Sykes plots to find the nature of nearest singularities which restricts convergence of the series. Validity of the series is further increased by Padé approximants which gives a converging sum for sufficiently large value of R.

3.2 Homotopy Analysis Method

To compare the obtained solutions we solve the problem by Homotopy analysis method. All perturbation techniques are based on small parameters so that at least one unknown must be expressed in a series of small parameters. Unlike perturbation methods, the HAM is independent of any small physical parameters. By using HAM we can transfer a non-linear problem into an infinite number of linear sub-problems. The HAM provides a convenient way to guarantee the convergence of series solution in conjunction with Padé sum, so that it is valid even if non-linearity becomes rather strong as compared to all other analytic methods.

Zeroth-order deformation problem

We seek solution of Eq.(9) by using HAM given by Liao (2004, 2012). We choose the base function to express $f(\lambda)$. The initial guess is written as,

$$f_0(\lambda) = -\frac{\lambda^3}{2(3\phi+1)} + \frac{3\lambda\phi}{3\phi+1} + \frac{3\lambda}{2(3\phi+1)} \tag{20}$$

and auxiliary linear operator is defined as,

$$L[f] = f'''' \tag{21}$$

The above linear operator satisfying the following property,

$$L[C_1\frac{\lambda^3}{6} + C_2\frac{\lambda^2}{2} + C_3\lambda + C_4] = 0$$

where C_1, C_2, C_3 and C_4 are constants to be determined. If $q \in [0,1]$ then the zeroth order deformation problem can be constructed as,

$$(1-q)L[f(n,q) - f_0(\lambda)] = qhH(\lambda)N[f(\lambda,q)] \tag{22}$$

subject to boundary conditions,

$$\begin{aligned} f(0,q) &= 0 \\ f(1,q) &= 1 \\ f'(1,q) &= -\phi f''(1,q) \\ f''(0,q) &= 0 \end{aligned} \tag{23}$$

where $0 \leq q \leq 1$ is an embedding parameter, h and H are non-zero auxiliary parameter and auxiliary function respectively. Further, N is a non-linear differential operator defined as,

$$N[f(\lambda,q)] = \frac{\partial^4 f(\lambda,q)}{\partial\lambda^4} + R\frac{\partial f(\lambda,q)}{\partial\lambda}\frac{\partial^2 f(\lambda,q)}{\partial\lambda^2} - R\frac{\partial^3 f(\lambda,q)}{\partial\lambda^3} \tag{24}$$

For $q = 0$ and $q = 1$, Eq.(22) has solution,

$$\begin{aligned} f(\lambda,0) &= f_0(\lambda) \\ f(\lambda,1) &= f(\lambda) \end{aligned} \tag{25}$$

As q varies from 0 to 1, $f(\lambda,q)$ varies from initial guess $f_0(\lambda)$ to exact solution $f(\lambda)$. By Taylor's theorem, Eq.(25) can be expressed as

$$f(\lambda,q) = f_0(\lambda) + \sum_{m=1}^{\infty} f_m(\lambda)q^m \tag{26}$$

where, $f_m(\lambda) = \frac{1}{m!}\frac{\partial^m f}{\partial q^m}\Big|_{q=0}$. Convergence of the above series (26) depends on the convergence control parameter h, which is chosen in such a way that (26) is convergent at $q = 1$. Then we have,

$$f(\lambda) = f_0(\lambda) + \sum_{m=1}^{\infty} f_m(\lambda) \tag{27}$$

$\mathbf{m^{th}}$-order deformation problem

Differentiating the zeroth order deformation problem equation (22) 'm' times with respect to q and lastly setting $q = 0$. The resulting m^{th} order deformation problem becomes,

$$L[f_m(\lambda) - \chi_m f_{m-1}(\lambda)] = hH(\lambda)\Re_m(\lambda) \tag{28}$$

and the homogeneous boundary conditions are,

$$f_m(0) = 0, \quad f_m(1) = 0, \quad f'_m(1) = -\phi f''_m(1), \quad f''_m(0) = 0 \tag{29}$$

where

$$\Re_m(\lambda) = f''''_{m-1} + R\sum_{n=0}^{m-1}[f'_n f''_{m-n-1} - f_n f'''_{m-n-1}] \tag{30}$$

and

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases} \tag{31}$$

We used Mathematica software to solve the system of linear equations (28) with appropriate boundary conditions (29) and obtain the solution as follows,

$$\begin{aligned} f_1(\lambda) = R\Bigg(&\lambda^7\left(\frac{3h\phi}{280(3\phi+1)^3} + \frac{h}{280(3\phi+1)^3}\right) + \lambda^3\left(-\frac{3h\phi}{40(3\phi+1)^3} - \frac{3h}{280(3\phi+1)^3}\right) \\ &+\lambda\left(\frac{9h\phi}{140(3\phi+1)^3} + \frac{h}{140(3\phi+1)^3}\right)\Bigg) \end{aligned} \tag{32}$$

$$\begin{aligned} f_2(\lambda) = \lambda^{11}&\left(\frac{3h^2R^2\phi^2}{30800(3\phi+1)^5} + \frac{h^2R^2}{92400(3\phi+1)^5} + \frac{h^2R^2\phi}{15400(3\phi+1)^5}\right) + \lambda^9\left(-\frac{3h^2R^2\phi^3}{560(3\phi+1)^5}\right. \\ &\left.-\frac{h^2R^2\phi^2}{160(3\phi+1)^5} - \frac{h^2R^2\phi}{420(3\phi+1)^5} - \frac{h^2R^2}{3360(3\phi+1)^5}\right) + \lambda^7\left(\frac{9h^2R^2\phi^2}{2800(3\phi+1)^5}\right. \\ &\left.+\frac{3h^2R^2\phi}{1960(3\phi+1)^5} + \frac{3h^2R^2}{19600(3\phi+1)^5} + \frac{27h^2R\phi^3}{280(3\phi+1)^5} + \frac{27h^2R\phi^2}{280(3\phi+1)^5}\right) \\ &+\lambda^7\left(\frac{9h^2R\phi}{280(3\phi+1)^5} + \frac{h^2R}{280(3\phi+1)^5} + \frac{27hR\phi^3}{280(3\phi+1)^5} + \frac{27hR\phi^2}{280(3\phi+1)^5}\right. \\ &\left.+\frac{9hR\phi}{280(3\phi+1)^5} + \frac{hR}{280(3\phi+1)^5}\right) + \lambda^3\left(\frac{9h^2R^2\phi^3}{140(3\phi+1)^5} + \frac{51h^2R^2\phi^2}{1400(3\phi+1)^5}\right. \\ &\left.+\frac{73h^2R^2\phi}{7700(3\phi+1)^5} + \frac{73h^2R^2}{107800(3\phi+1)^5} - \frac{27h^2R\phi^3}{40(3\phi+1)^5} - \frac{153h^2R\phi^2}{280(3\phi+1)^5}\right) \\ &+\lambda^3\left(\frac{-39h^2R\phi}{280(3\phi+1)^5} - \frac{3h^2R}{280(3\phi+1)^5} - \frac{27hR\phi^3}{40(3\phi+1)^5} - \frac{153hR\phi^2}{280(3\phi+1)^5}\right. \\ &\left.-\frac{39hR\phi}{280(3\phi+1)^5} - \frac{3hR}{280(3\phi+1)^5}\right) + \lambda\left(-\frac{33h^2R^2\phi^3}{560(3\phi+1)^5} - \frac{2063h^2R^2\phi^2}{61600(3\phi+1)^5}\right. \end{aligned}$$

$$-\frac{703h^2R^2\phi}{80850(3\phi+1)^5}-\frac{703h^2R^2}{1293600(3\phi+1)^5}+\frac{81h^2R\phi^3}{140(3\phi+1)^5}\Bigg)+\lambda\Bigg(\frac{9h^2R\phi^2}{20(3\phi+1)^5}$$
$$+\frac{3h^2R\phi}{28(3\phi+1)^5}+\frac{h^2R}{140(3\phi+1)^5}+\frac{81hR\phi^3}{140(3\phi+1)^5}+\frac{9hR\phi^2}{20(3\phi+1)^5}$$
$$+\frac{3hR\phi}{28(3\phi+1)^5}+\frac{hR}{140(3\phi+1)^5}\Bigg) \tag{33}$$

Convergence of HAM

The series (27) contains the auxiliary parameter h which is known as convergence control parameter and it influences the convergence rate and region of the series. To ensure that this series converges, we need to choose proper value for h. To obtain the permissible ranges of the parameter h, h-curves are plotted (Fig. 9).

4. Results and Discussion

The problem of laminar flow in a channel of porous walls with velocity slip is studied using two novel semi-numerical and Semi-analytical methods:computer extended series method and Homotopy analysis method (HAM). The motion of fluid is governed by a nonlinear ordinary differential equation (9) with boundary conditions (10).

Using recurrence relation and MATHEMATICA we generate large number ($n = 30$) of universal polynomial functions $f_n(\lambda)$ for different slip coefficients ϕ. This enables one in obtaining large number of universal polynomial functions $f_n(\lambda)$ for different slip coefficient ϕ. The series (14)-(15) representing velocity profiles are analyzed using Padé approximants for different Reynolds number R and also the effect of slip coefficient on these profiles are shown in figures 3, 4 and 6. Domb-Sykes plot (Fig. 2) shows the singularity restricting convergence of the series representing velocity profiles.

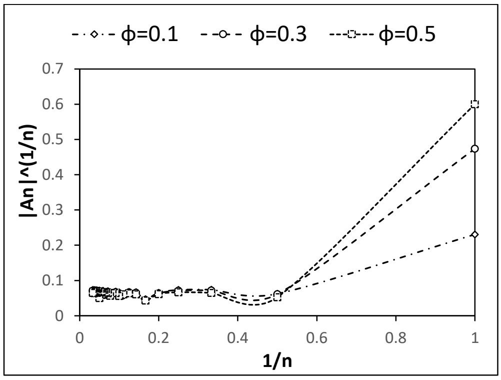


Figure 2: Domb-Syke plot for velocity profiles

Figures 3 and 4 show influence of the slip velocity on the channel flow with suction and injection. The no slip case corresponds to $\phi = 0$. It is observed that the slip velocity is more sensitive to changes in the slip coefficient for small values and seems to approach an asymptotic value for large slip coefficient, that is, increasing slip leads to flattening of profiles and reduced wall shear stresses. Also, increasing suction results in flatter profiles and velocity profiles restricted to thinner boundary layer. Whereas, fluid injection does not result in marked changes in the velocity profiles but decreases the wall shear rate. The influence of normalized slip velocity for various values of R are shown in Figure 5, which shows U_s increases with ϕ and approaches asymptotic values.

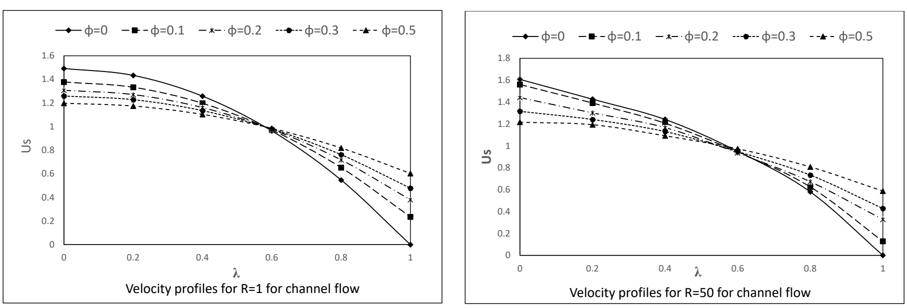
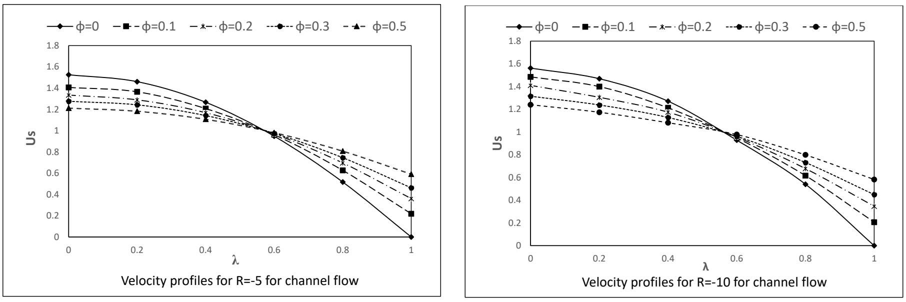


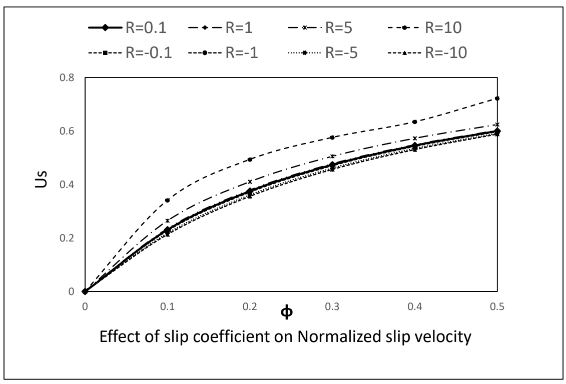
Figure 3: Velocity profiles for channel flow with suction



Velocity profiles for R=-5 for channel flow

Velocity profiles for R=-10 for channel flow

Figure 4: Velocity profiles for channel flow with injection



Effect of slip coefficient on Normalized slip velocity

Figure 5

Figure 6 is a plot of dimensionless transverse velocity with λ, shows the effect of axial slip coefficient on the mid channel transverse velocity profile. Large suction rate results in linear transverse velocity profiles whereas injection make only small deviations in the shape.

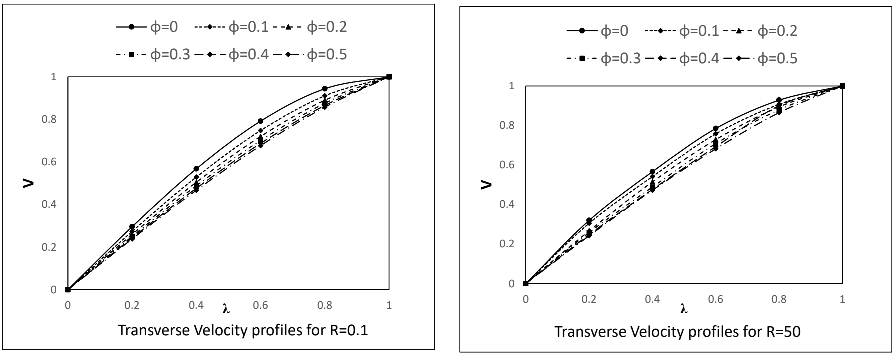


Figure 6: The effect of slip coefficient on dimensionless transverse velocity for $Re = 1000$

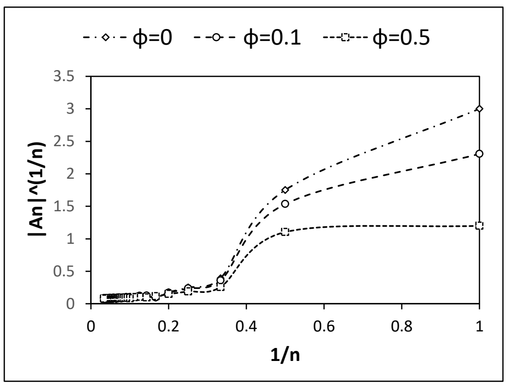
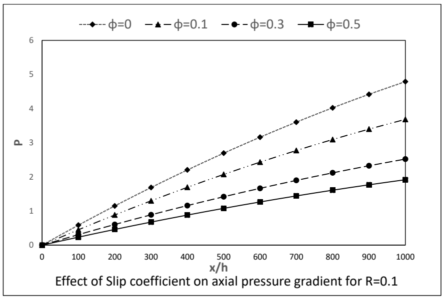


Figure 7: Domb-Syke plot for pressure gradient

The coefficients a_n of the series (18) represents the pressure gradient P are decreasing in magnitude, but have no regular sign pattern. Domb-Syke plot (7) after extrapolation, confirms the radius of convergence of series (18) to be $R = 12.27, 12.40, 14.24$ (with an error of 10^{-5}) for different slip coefficients ($\phi = 0, 0.1, 0.5$) respectively. Direct sum of the series is valid only up to $R < 12$. We obtained the results of P for sufficiently large R (up to 50) using Padé approximants (Bender and Orszag (1987)) as compared to earlier findings.

Figure 8 shows that the magnitude of pressure gradient P, increases with $\frac{x}{h}$ for fixed entrance $Re = 1000$. Inspection of figure reveals that the values of P decrease with the influence of slip coefficient ϕ, at the porous wall which leads to reduction in the shear stress at the membrane surface. This leads to the fact that, the effect of velocity slip at the porous binding wall is to decrease magnitude of the pressure gradient. Another interesting trend in evidence in the graph is that an increase in R will result in decrease of shear stress which in-turn reduces P.



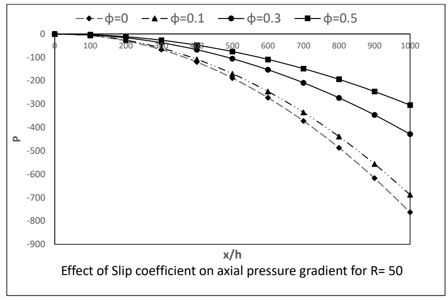
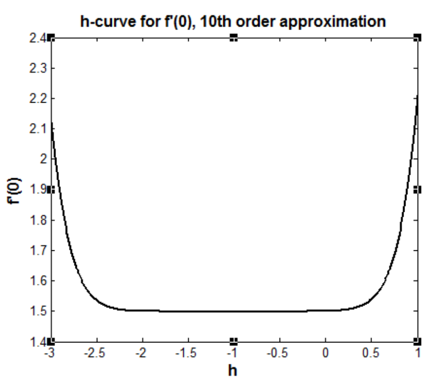


Figure 8: Variation in pressure gradient as a function of slip coefficient ϕ

To confirm certainty, to validate accuracy and efficiency of the results obtained by Computer extended Series method , the problem is also analyzed by an elegant homotopy analysis method (HAM) in conjunction with Padé sum to accelerate convergence of the series. We plot h-curves to find the convergence range and also the rate of approximations for the series representing $f'(0)$ and P when $R = 0.1, \phi = 0$ respectively from 10^{th} order HAM approximations. The range for admissible values of h for different values of R and ϕ is different. From the figure 9 it is observed that series representing $f'(0)$ and $f'''(1)$ are convergent when $-2.2 \leq h \leq -0.3$.



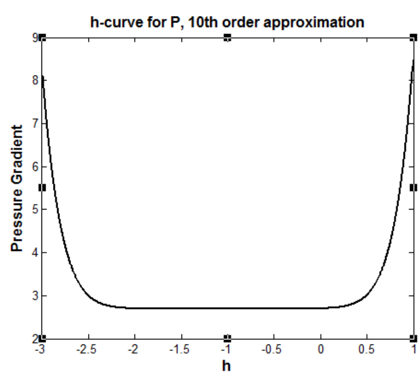


Figure 9: h-curves for 10th order approximations

5. Concluding Remarks

In the present work, we employed two semi-numerical methods, known as Computer Extended Series method and Homotopy Analysis Method to solve fourth order non-linear differential equation modeling laminar flow through porous channels. The effect of non-zero tangential slip velocity on velocity field and pressure gradient are analyzed. The study confirms that the proposed methods converges to the solution for moderately large values of Reynolds number as compared to the earlier findings. The results obtained were compared and an excellent agreement was observed.

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